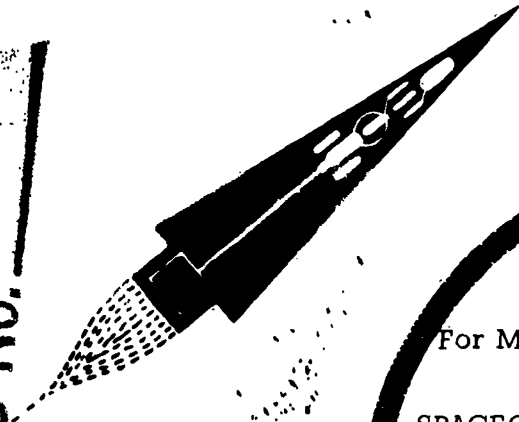
SPACE POWER AND PROPULSION SECTION



THIRD QUARTERLY
STATUS REPORT

For Month Ending December 31, 1962

SPACECRAFT ELECTRIC GENERATING AND
PROPULSION SYSTEM INTEGRATION

Under Contract AF33(657)-8488

For

AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE
OHIO

GENERAL ELECTRIC
CINCINNATI 15, OHIO

SPACE POWER AND PROPULSION SECTION

THIRD QUARTERLY STATUS REPORT

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SPACECRAFT ELECTRIC GENERATING AND PROPULSION SYSTEM INTEGRATION

CONTRACT NUMBER AF33(657)-8488

RE-ENTRY SYSTEMS DEPARTMENT
GENERAL ELECTRIC COMPANY
CINCINNATI 15, OHIO

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I. INTRODUCTION - SUMMARY OF THIRD QUARTER PROGRESS

This report summarizes the work accomplished during the third quarter of the Spacecraft Electric Generating and Propulsion System Integration Study Program under Air Force Contract No. AF33(657)-8488. The primary objectives of this program are:

1. To develop, demonstrate, and provide a working manual and digital computer program for the system design analysis and optimization procedure known as LEADER.

2. To apply this procedure to space power-propulsion systems employing the SPUR 350 KW and 1 MW nuclear space power plants.

The initial phase of the contract effort has been devoted to the solution of two illustrative problems, (1) optimization of a nuclear space power plant-transmission line system, and (2) optimization of a space power plant condensing radiator panel, as a means for satisfying the first objective. The previous quarterly progress report contained the results of the model formulation analyses of the two illustrative problems and the results of the optimization of the transmission line problem. The optimization results were obtained, however, with a transmission line model which lacked certain of the parameter interaction terms. This has been rectified during the third reporting period and revised optimization results have been obtained. The radiator panel optimization has, in addition, been completed. The results obtained for the two illustrative problems contain

(1) the values of each of the independent design parameters which minimize the over-all system weight within the boundaries and constraints specified, (2) the value of the minimum over-all system weight, (3) the boundaries or constraints which prevent further reduction in over-all system weight, and (4) the sensitivity of the over-all system weight to small changes in the optimum design parameters.

The process of solving the two illustrative problems has served to clarify the capabilities of the two computer programs comprising LEADER and to identify the various procedural elements which must be performed in order to achieve effective utilization of the LEADER technique. These procedural elements are being described in a User's Manual in order to facilitiate the application of LEADER to subsequent problems. During the subject reporting period, the first part of this User's Manual has been completed and is, therefore, included as a supplement to this progress report. This section contains the procedures recommended for the model formulation phase of LEADER problems and includes whatever tabular data may be needed. The results of the model formulation phase will be a series of analytical models of each of the sub-systems comprising the over-all system under investigation. These models will then be assembled into an over-all system model and processed by the optimization program. The second section of the User's Manual will contain the procedures necessary for this final phase of the analysis. This section has been outlined and detailed definition is in progress. It is expected that this second section of the manual will be completed within the next few weeks.

The process of generating and accumulating basic design data on the major sub-systems for the 350 KW arc jet and the 1 MW ion engine power-propulsion systems has been continued during this reporting period. Design procedures and performance characteristics have been established for the turbine, and the generator sub-systems. These procedures will be described in more detail in the following sections. The task of formulating the analytical model of these sub-systems has been initiated. It is expected that the entire model formulation process will be completed by the second week of February.

II. ILLUSTRATIVE PROBLEMS

The two illustrative problems have been structured to serve as a vehicle for:

(a) developing the procedural elements of the LEADER technique which would be required to supplement the existing computer programs,

(b) as a demonstration of the type of information required for input and the type of results to be expected from the use of LEADER on complex system design problems, and

(c) as a guide in the development of a User's Manual which will facilitate the use of LEADER for such problems.

The mechanics of the model formulation process were discussed in relation to the two illustrative problems in the previous progress report and will, therefore, be omitted from this discussion. This section will, instead, report on the process evolved for feeding the system model into the optimization program and on the type of results obtained. It should be noted that the number of independent variables and non-linear constraints required for each of the problems have been reduced considerably from the previous formulations. These reductions reflect a substantial improvement in our understanding of the characteristics and capabilities of the optimization program and result in a comparable reduction in the effort required for this process.

A. Transmission Line Problem

The transmission line model has been revised and simplified by the elimination of all equality constraints. The parametric constraint on power has been replaced by an input constant - P_1 - and the remaining constraint by an auxiliary equation. In addition, the voltage has been eliminated as a variable and replaced by a constant of 100,000 volts. The net effect has been a reduction of the model from a system of 7 independent variables with 4 inequality constraints (each equality constraint was expressed as a pair of inequality constraints) to a system of 4 independent variables with no constraints. The resulting model is illustrated in Table 1.

The above revised model was reprocessed by the optimization program in order to correct the results provided in the previous report which had been obtained with several of the parameter interaction terms omitted. The results obtained are illustrated in Table 2. These results indicate somewhat larger transmission line lengths and correspondingly higher line loss factors, but do not differ appreciably from the results obtained with the previous formulation.

Influence coefficients have been obtained to indicate the sensitivity of the weight to changes in the design parameters from their optimum values. The influence coefficients have been defined as the increase in total system weight resulting from a 1% increase in each of the design variables. Results are contained in Table 3. The negative signs for the temperature, frequency and voltage influence coefficients indicate that further reductions in system weight can be obtained by increasing these parameters beyond their maximum values. If these maximum values reflect a state-of-the-art limitation, as is the case with the temperature limit for example, the influence coefficient provides a measure of the return to be achieved from a development program structured to raise this limit.

B. Radiator Panel Problem

The radiator panel model has been similarly revised and simplified. The equality constraint equations have been removed and replaced by equivalent auxiliary equations and the variables defined by these equations eliminated from the list of independent parameters. The net effect of this process is that bounds on these auxiliary variables are removed. Although this did not introduce any complications in the transmission line problem, the radiator optimization could not achieve a realistic design without some additional constraints. It was necessary, therefore, to identify and use these inequality constraints to replace the constraints applied by the former bounds on the auxiliary parameters.

The transform for normalizing temperature to the 0 to 1 range was retained. The transforms for the remaining independent variables were, however, replaced by a simple multiplier which would set the maximum value of each normalized variable to 1 but allow the minimum value to float. The net effect of this approach is to convert the standard transform equation:

$$x = aX + b$$

to the form: $x = aX$

which results in a substantial reduction in the amount of manual calculations required for converting the results of the model development analysis to the form required for the optimization program.

The net effect has been the reduction of the radiator model from a system of 10 independent variables with 10 inequality constraints (5 pairs of equality constraints) to a system of 3 input parameters, 4 independent variables, and 3 inequality constraints. The resulting model is illustrated in Table 4.

Six computer optimization runs were made with this model in order to explore the characteristics of the radiator panel. The results of these runs are summarized in Table 5. Runs 1 and 2 were made with a panel heat capacity of 150 KW thermal and a fin effectiveness of 70% but with different initial temperatures. The results would seem to indicate that the temperature effect is not strong enough to indicate a single optimum design. The probability of achieving 10,000 hours of operation without a single meteorite puncture is also indicated. Note that this survival probability increases with increased temperature and that the ambiguity with respect to temperature would have been eliminated if the design had been constrained to a single value of survival probability. This, however, was not done in order to simplify the illustrative problem.

A comparison of runs 2, 3, and 4 indicate the trend in panel weight and in survival probability as the thermal heat capacity of the panel is increased with both fin effectiveness and panel temperature held constant. It is apparent that the temperature should be increased with increased heat capacity in order to maintain a given level of survival probability.

Runs 5 and 6 illustrate the effects of a variation in fin effectiveness at constant heat capacity and panel temperature. These results indicate a substantial variation in panel weight but only a relatively small effect on survival probability.

Although the above data is insufficient to completely assess the trade-off's between panel temperature and fin effectiveness it is quite apparent that these two parameters are the most significant variables in establishing an optimum panel weight at a fixed level of survival probability. It is equally apparent that this trade-off could be completely resolved by either the calculation of three to six runs or by the augmentation of the radiator model by an additional constraint on the survival probability.

Table 6 contains a summary of influence coefficients for each of the six computer runs. These were obtained by increasing the normalized values of each of the independent variables by 1%. The actual change involved is indicated in parentheses. Note that all of the influence coefficients are negative indicating that an increase in any of the independent variables will result in a decrease in radiator panel weight. The tube spacing would appear to be the most sensitive design variable over the range of variation investigated.

All of the six runs were forced up against the third of the three constraints. This limit is imposed by a requirement for an armour thickness greater than .1 inches. The effect of temperature on this constraint formulation was, however, ignored in order to simplify the expression. This resulted in the observed armour thickness of .143 inches at the high temperature levels of runs 2, 3, and 4. The fact that all optimization runs ended on this constraint indicates that the more rigorous expression should have been included.

III. LEADER USER'S MANUAL

The LEADER User's Manual will contain a summary of the procedural elements of the LEADER technique with particular emphasis on those manual operations required for the preparation of system and sub-system data for the model development and optimization computer programs and for the evaluation of the results obtained. It will consist of two major sections - one on model development procedures and the other on system assembly and optimization procedures. The section on model development procedures has been completed and is included as a supplement to this progress report. The section on system assembly and optimization is in progress and is expected to be completed within the next few weeks. A brief discussion of the approach to be used in this section is included in the following. The computer manual will be re-issued as one of the technical summary reports summarizing the work performed under this study contract.

The first step in the procedure is the identification of the objective function - the parameter to be optimized - and of the over-all system constraints to be imposed. The individual sub-system models will then be analyzed in order to identify those quantities required as inputs to each model and those quantities that will be available as outputs. The sequence of model calculations must be established in order to ensure that sub-systems are not calculated until the required inputs are available.

Relationships which link two or more subroutines must then be added. This will include such considerations as heat balances, energy balances, flow continuity, speed matching, etc. Each of these relationships that apply can be used to eliminate one independent variable. The variables to be eliminated should be selected with care since they will become auxiliary variables which cannot be maintained within specified boundaries by the optimization process unless specific constraint equations are provided.

All system and sub-system constraints must be identified. Equality constraints must be treated in a fashion similar to the equations defining the auxiliary variables and will, consequently, permit the elimination of an additional independent variable. Inequality constraints may be imposed but should be expressed as functions of the remaining independent variables. The ranges of interest of the independent variables must be identified and the variables normalized to a common range. The model and constraint equations must be modified to receive the normalized variables.

The resulting equations must then be prepared for insertion into the optimization program. All expressions which deviate from the standard polynomial format will involve special handling in order to get them into the optimization program. Finally, a set of starting values must be calculated which will satisfy all of the system constraints imposed.

The system can then be run on the computer and the optimum design obtained. The computer results will, in addition, indicate the particular constraints which are in operation at the resulting optimum design and the sensitivity of the objective function to small deviations of the independent variables from the optimum design.

The above procedures are being described in considerably more detail in the second section of the User's Manual which is in preparation. These descriptions are, in addition, being implemented by examples which will aid in illustrating each specific operation. This section will then be combined with the existing section on model development procedures and the report re-issued as a technical summary report. The computer program listings will be included as appendices to this report in order to facilitate the utilization of the overall technique.

IV. SPUR MODEL FORMULATION

Effort has been continued on the formulation of the 350 KW arc jet and the 1 MW ion engine power-propulsion system of SPUR during this reporting period. These efforts have been associated previously with the accumulation of basic design data and performance characteristics, with the development of sub-system design procedures, and with the identification of appropriate design criteria and assumptions. This process has been completed for the turbine and the generator sub-systems and is expected to be completed for the remaining sub-systems within the next few weeks.

The results of these analyses have been used to develop preliminary model formulations for the turbine and generator sub-systems. These formulations serve as a summary of the results of the respective design analyses and as an indication of the additional analysis required to reduce the design procedure to the form required by the optimization program. The preliminary turbine and generator formulations will be described in the following section along with the design approaches and assumptions upon which they are based.

The overall approach has been based upon the identification of sub-system weight as the objective function. The individual sub-system weights will then be summed and the overall weight minimized by the optimization process. This minimization will be carried out subject to constraints on overall system length and diameter required for integration with the Saturn C-1B boost vehicle.

A. TURBINE SUB-SYSTEM

The turbine design philosophy had been based upon the use of conventional steam turbine practice employing full admission-root impulse stages. The following design assumptions have been made:

1. The flow of saturated vapor can be represented with sufficient accuracy by the use of perfect gas formulae when suitable values of the gas constant and the specific heat ratio are employed.

2. The total to total efficiency of a wet vapor turbine can be represented as a function of the average stage pitch line velocity ratio U_p/C_o (where U_p in the pitch line velocity and C_o is the velocity associated with converting the total enthalpy drop across the stage to a kinetic energy), the interstage moisture removal effectiveness, and the tip clearance to bucket span ratio. The data of Figure 1 illustrates the effect of the velocity ratio and the moisture removal factor. The efficiency obtained from Figure 1 is corrected by subtracting the quantity:

   $$3 \text{ (Clearance to Span Ratio)}$$

   This correlation assumes a high level of design and manufacturing precision on bucket profiles and interstage labyrinth seals and has been obtained from a large body of steam turbine test data.

3. Turbine discs are of the constant stress type.

4. Refractory metal alloys are used in turbine rotor construction which permit the use of design stress limits for 0.1% creep after 10,000 hour operation as shown by the data of Figure 2.

5. Untapered buckets which are integral with the wheel have been used.

A schematic drawing of the turbine is included in Figure 3. Table 7 contains a summary of the preliminary turbine model developed which has been arranged according to the following format:

1. Input - The independent variables which establish the turbine design in sufficient detail to permit an estimation of its weight. These parameters will, in general, be optimized by the LEADER technique unless they are obtained as auxiliary outputs from the other sub-system models.

2. Auxiliary Equations - The dependent variables which are needed for the weight calculation, for establishing the sub-system constraints, or as inputs to the other sub-system models.

3. Constraint Equations - The physical constraints imposed by turbine design considerations. These constraints must be augmented by linear constraints and by additional non-linear constraints. The linear constraints are the bounds that are established on the independent variables. The non-linear constraints are introduced, where needed, in order to establish bounds on the dependent variables. These must be expressed in terms of two or more independent variables.

4. Objective Function - A summary of the individual turbine element weights which will be added to the weights of the other sub-systems and minimized.

5. Parameters - Those factors which will be assumed to be constant during any optimization run. Any of these parameters could be varied in subesequent optimization runs or transferred to the input section where they would be treated as additional independent variables.

6. Empirical Relations - Empirical equations required for the evaluation of the auxiliary variables which must be developed by the techniques described in the LEADER User's Manual. Note that the turbine equations required are functions of either one or two variables for each working fluid selected.

7. Auxiliary Outputs - Parameters which are expected to be required as input to other sub-system models. This list will be augmented as the other model formulations are developed.

The above format has been utilized as an aid in identifying the interfaces between the turbine sub-system and the other sub-systems and in identifying the specific variables which will be optimized by the LEADER technique. It is particularly important to keep a running count on the number of independent variables as a total in excess of 50 would require the overall system model to be treated as two or more sub-optimization problems.

B. GENERATOR SUB-SYSTEM

The generator sub-system model has been developed for a radial gap generator. A similar model is in process for an axial gap generator. The radial gap design procedure has been based upon the following assumptions:

1. The electromagnetic design is limited by values of:

   15 for the ratio of rotor tooth depth to flux gap length and
   30 for the ratio of rotor tooth gap at the outside of the tooth to flux gap length.

2. Corresponding to the above limits, the stator to rotor leakage flux has been assumed to be 20% of the working gap flux.

3. The effective A.C. flux is 1/2.2 times the working gap flux.

4. The pole embrace is 2/3.

5. Rotor hot spots are within 50° of the coolant temperature, and, consequently, the rotor stress limit can be expressed as a function of the coolant outlet temperature. Assumed values are illustrated in Figure 4.

6. The field coil proportions have been selected to correspond with minimum frame weight per coil cross sectional area and mean diameter.

7. Stator stack and conductor weights have been calculated by applying a correction factor to account for the end turns and end turn potting to an armature weight based upon a solid lamination material

A schematic drawing of the generator is included in Figure 5. Table 8 contains a summary of the preliminary generator model developed according to the same format used for the turbine.

TABLE 1

Transmission Line Model

A. Variables

Parameter	Symbol	Uncoded Bounds		Coded Ind. Variable	Transform Equations
Inverse Efficiency - %	$1/\eta$	1.0101	1.260	x_1	$1/\eta = 1.0101 + 25\,x_1$
Frequency - cps	f	400	3200	x_2	$f = 400 + 2800\,x_2$
Temperature - °R	T	2000	2500	x_3	$T = 2000 + 500\,x_3$
(Line Length)^2 - ft	l^2	$(100)^2$	$(100{,}000)^2$	x_4	$l^2 = 10^4 + 10^{10}\,x_4$

B. Input

Power	Coded Parameter - P_1
350 KW	0
1000 KW	.3023
2500 KW	1.0000

C. Auxiliary Equations

$$G_1 = \frac{.96 - x_1}{.95 + 23.75\,x_1} \qquad \left(\frac{1}{1-\eta}\right)$$

$$G_2 = \frac{1}{30}\ln\left(10^4 + 10^{10}\,x_4\right) \qquad (\ln l)$$

$$G_3 = .1\ln(P_1) \qquad (\ln P)$$

$$G_4 = \frac{1}{.3}\ln(1.01 + .25\,x_1) \qquad (-\ln \eta)$$

TABLE 1 (Continued)

Transmission Line Model

D. Objective Function

$$
\begin{aligned}
W = 12{,}277.2 &+ 19{,}135.5\,P_1 + 4741.5\,P_1x_1 - 2423.6\,P_1x_2 - 1846.3\,P_1x_3 \\
&+ 689.2\,P_1x_4 - 599.9\,P_1x_1x_2 - 457.0\,P_1x_1x_3 \\
&+ 170.6\,P_1x_1x_4 + 13{,}094.4\,P_1x_4G_1 \\
&+ 771.9\,x_1 - 97.7\,x_1x_2 - 74.4\,x_1x_3 + 27.8\,x_1x_4 + 527.6\,x_1x_4G_1 \\
&- 769.5\,x_2 \\
&- 3400.6\,x_3 \\
&+ 112.2\,x_4 + 2131.6\,x_4G_1 \\
&- 7635.4\,G_2 + 2306.4\,G_2G_3 - 69.2\,G_2G_4 + 2616.6\,G_2{}^5 \\
&- 1495.8\,G_2{}^5G_3 + 448.7\,G_2{}^5G_4 \\
&+ 4317.4\,G_3 \\
&+ 129.5\,G_4
\end{aligned}
$$

E. Constraints

None

TABLE 2

OPTIMUM OVER-ALL SYSTEM DESIGN

Transmission Line Model

Power - P	KW	350	1000	2500
Efficiency - η	%	94.20	96.48	93.74
Frequency - f	cps	3199.8	3055.2	3058.6
Temperature - T	°R	2500.0	2500.0	2500.0
Length - l	ft	71,880	44,090	50,100
Voltage - E	Volts	10^{10}	10^{10}	10^{10}
Loss Factor - $1-\eta$	%	5.80	3.52	6.26
Total System Weight - W	lbs	4,845.3	10,179	22,174
Specific Weight - W/P	lbs/KW	13.84	10.18	8.86

TABLE 3

INFLUENCE COEFFICIENTS

Transmission Line Model

Power Output	KW	350	1000	2500
Total System Weight	lbs	4,845.3	10,179	22,175
Influence Coefficients - lbs				
Power		168.6	169.1	170.3
Efficiency		.111	67.50	123.5
Temperature		-36.36	-39.89	-53.88
Frequency		-9.41	-19.42	-32.09
Line Length		-1.90	.8000	1.00
Voltage		-1.572	-1.733	-6.34022

TABLE 4

Radiator Panel Model

A. Variables

Parameter	Symbol	Uncoded Bounds		Coded Ind. Variable	Transfer Equation
Fin Thickness - in	t	.10	.30	x_1	$t = .3x_1$
Tube Spacing-ft^{-1}	N/L	0	66.64	x_2	$N/L = 66.64x_2$
Tube Diameter - in	d	.15	.50	x_3	$d = .5x_3$
(Temperature)^2-(°R)^2	T^2	$(1650)^2$	$(1850)^2$	x_4	$T^2 = (1650)^2(1 + .2571162x_4)$

B. Input

Runs	P_1 Heat Capacity BTU/hr.	P_2 Stefen-Boltzman Constant	P_3 Emissivity	P_4 Conductivity	P_5 Fin Effectiveness	P_6 Fin Parameter
1	511,950 (150 KW)	$1.713\ (10)^{-9}$	.9	52	.70	.41
2	1,023,900 (300 KW)	$1.713\ (10)^{-9}$	.9	52	.70	.41
3	1,535,850 (450 KW)	$1.713\ (10)^{-9}$	.9	52	.70	.41
4	1,535,850 (450 KW)	$1.713\ (10)^{-9}$	.9	52	.80	.20
5	1,535,850 (450 KW)	$1.713\ (10)^{-9}$	.9	52	.90	.08

TABLE 4 (Continued)

Radiator Panel Model

C. Auxiliary Equations

G_1	$= \sqrt{x_4}$	(T)
G_2	$= \dfrac{P_1}{3413}\left[.7 - 3.6(10)^{-4}G_1\right]$	(Vapor Volume Flow - V, cps)
G_3	$= \sqrt{G_2}$	
G_4	$= \dfrac{.306\ G_2}{x_3^{\ 2}}$	(No. of tubes - N)
G_5	$= \dfrac{P_4 P_6 x_1}{2P_2 P_3 G_1^{\ 3}}$	(Fin length - l, in)
G_6	$= \dfrac{6}{x_2} - \dfrac{x_3}{2} - G_5$	(Armour thickness - δ, in)
G_7	$= .958G_3$	(Header diameter - D, in)
G_8	$= \dfrac{G_4}{x_2}$	(Panel length - L, ft)
G_9	$= P_2 P_3 G_8 G_1^{\ 4}$	($\sigma \epsilon L T^4$)
G_{10}	$= \dfrac{6P_1 - G_9\ (G_7 + 2G_6)}{G_9\ x_2\ (x_3 + 2G_6 + 2G_5 P_5)}$	(Panel width - h, ft)
G_{11}	$= \dfrac{1}{G_{10}}$	

TABLE 4 (Continued)

Radiator Panel Model

D. Objective Function

$$
\begin{aligned}
W = G_{10} \Big[9.65\, x_1 G_8 & - .804\, x_1 x_3 G_4 - 1.608\, x_1 G_4 G_6 \\
& + .1247\, x_2 x_3 G_8 + .3731\, x_2 G_6 G_8 - .0491\, x_2 G_8 \\
& + 3.736\, x_3 G_6 G_8 - .7518\, x_3 G_8 + .2205\, G_3 G_6 G_8 - .0088\, G_3 G_8 \\
& + .2474\, G_3 G_8 G_{11} - 3.40 G_6 G_8 + 2.066\, G_6^2 G_8 + 5.478\, G_6 G_8 G_{11} \\
& + .4556\, G_8 - .4367\, G_8 G_{11} \Big]
\end{aligned}
$$

E. Constraints

1. $557{,}697\, x_2 x_3^2 > .38577\, P_1 - 1.02857\, (10)^{-7}\, P_1\, x_4$
2. $6 > .5\, x_2 x_3 + .1\, x_2$
3. $36 - 1.2\, x_2 + .01\, x_2^2 - 6\, x_2 x_3 + .1\, x_2^2 x_3 + .25\, x_2^2 x_3^2 > 2.67\, P_6 x_1 x_2^2$

TABLE 5

Optimum Radiator Panel Design

Run No.			1	2	3	4	5	6
Heat Capacity	Q	KWT	150	150	300	450	450	450
Fin Effectiveness	η_f		.70	.70	.70	.70	.80	.90
Heat Transfer Coefficient	2		.41	.41	.41	.41	.20	.08
Temperature	T	°R	1671	1754	1754	1753	1678	1678
Fin Thickness	t	in	.120	.148	.149	.150	.114	.114
Tube Diameter	d	in	.376	.256	.253	.252	.382	.382
Tube Spacing	N/L	ft	8.35	8.49	8.49	8.48	10.28	12.59
Vapor Volume Flow	V	cfs	14.76	10.26	20.62	31.02	43.24	43.17
Number of Tubes	N		31.9	48.2	98.6	149.9	90.9	90.4
Fin Length	l	in	.422	.435	.438	.439	.285	.180
Armour Thickness	δ	in	.108	.144	.143	.143	.107	.105
Header Diameter	D	in	3.68	3.07	4.54	5.34	6.30	6.29
Panel Length	L	ft	3.82	5.65	11.62	17.65	8.84	7.18
Panel Width	h	ft	6.36	3.46	3.24	3.10	7.29	8.53
Panel Area	A	ft^2	48.6	39.0	75.2	109.4	128.8	122.4
Panel Vulnerable Area	A_v	ft^2	23.6	15.7	35.0	55.3	80.8	88.9
Survival Probability in 10,000 hrs.	P	%	87.3	96.1	91.4	86.8	62.8	59.9
Radiator Panel Weight	W	lbs	42.4	43.2	94.9	151.8	127.1	118.6

TABLE 6

INFLUENCE COEFFICIENTS

Radiator Panel Model

Run No.		1	2	3	4	5	6
Panel Heat Capacity - KWT		150	150	300	450	450	450
Fin Effectiveness		.70	.70	.70	.70	.80	.90
Panel Temperature - °R		1671	1754	1754	1753	1687	1687
Influence Coefficients - lbs							
Fin Thickness - in	(Δ = .003 in)	-.0022	-.0124	-.015	-.0035	-.066	-.048
Tube Spacing - ft^{-1}	(Δ = .67 ft^{-1})	-.56	-.64	-1.28	-2.08	-1.70	-1.35
Tube Diameter - in	(Δ = .005 in)	-.057	-.040	-.171	-.024	-.140	-.137
Temperature - °R	(Δ = 2.1° R)	-.024	-.018	-.081	-.0096	-.079	-.060

TABLE 7

Preliminary Turbine Sub-System Model

A. Input

1. Turbine Inlet Total Temperature - T_{t1} - °R
2. Turbine Inlet Total Pressure - P_{t1} - psia
3. Mass Flow Rate - W - lb/sec
4. Nozzle Angle - α
5. First Stage Bucket Tip Radius - r_{t1} - in
6. First Stage Nozzle Exit Velocity - C_1 - fps
7. Leaving Loss Factor - LL
8. Moisture Removal Parameter - M
9. Clearance - CL - in
10. Turbine Discharge Total Temperature - T_{t2} - °R

B. Auxiliary Equations

1. $V_1 = \dfrac{C_1}{\sqrt{\gamma g R T_{t1}}}$ (First Stage Nozzle Velocity Function)
2. $FF_1 = f_1 \left[V_1\right]$ (First Stage Flow Function)
3. $A_1 = \dfrac{P_{t1}\,(FF_1)\,\sin\alpha}{W\sqrt{RT_{t1}}}$ (First Stage Annulus Area)
4. $r_{h1} = \sqrt{1 - \dfrac{A_1}{\pi r_{t1}^{2}}}$ (First Stage Rotor Hub Radius)
5. $U_{h1} = .5C_1$ (First Stage Hub Velocity)
6. $N = \dfrac{30\,U_{h1}}{\pi r_{h1} r_{t1}}$ (Rotor Speed)
7. $\sigma_{b1} = 4.52\,A_1 \rho_b \left(\dfrac{N}{1000}\right)^2$ (First Stage Bucket Stress)

TABLE 7 (Continued)

Preliminary Turbine Sub-System Model

B. Auxiliary Equations (Continued)

8.	F_m	=	f_2 [Working Fluid, T_{t1}, T_{t2}]	(Ratio of Ideal Rankine to Carnot Efficiency)
9.	H_v	=	f_3 [Working Fluid, T_{t1}]	(Heat of Vaporization)
10.	ΔH_t	=	$.85\ H_v F_m\ [1 - T_{t2}/T_{t1}]$	(Nominal Enthalpy Drop)
11.	C_2	=	$\sqrt{2gJ\Delta H_t LL}$	(Leaving Velocity)
12.	V_2	=	$C_2/\sqrt{\gamma g R T_{t2}}$	(Last Stage Velocity Function)
13.	P_{t2}	=	$f_4\ [T_{t2}]$	(Total Discharge Pressure)
14.	FF_2	=	$f_1\ [V_2]$	(Last Stage Flow Function)
15.	A_2	=	$[P_{t2}\ (FF_2)] / [W\sqrt{RT_{t2}}]$	(Last Stage Annulus Area)
16.	U_{h2}	=	$.5C_2 / \sin\alpha$	(Last Stage Hub Velocity)
17.	r_{h2}	=	$[30\ U_{h2}] / [\sqrt{\pi A_2 N^2 + 900\ U_{h2}^{\ 2}}]$	(Last Stage Hub Radius)
18.	r_{t2}	=	$\dfrac{30U_{h2}}{\pi N r_{h2}}$	(Last Stage Tip Radius)
19.	$\Sigma U_h^{\ 2}$	=	$.5\ gJ\Delta H_t$	
20.	n	=	$[2\Sigma U_h^{\ 2}] / [U_{h1}^{\ 2} + U_{h2}^{\ 2}]$	(No. of Stages)
21.	L	=	2.5 n S	(Rotor Length)
22.	F_1	=	$f_5\ (T_{t1})$	(First Stage Allowable Stress)

TABLE 7 (Continued)

Preliminary Turbine Sub-System Model

B. Auxiliary Equations (Continued)

23.	t_{n1}	$= [\sigma_{b1} S]/[2 F_1]$	(First Stage Wheel Neck Thickness)
24.	t_{h1}	$= t_{n1}\, e^{\{[\rho_b U_{h1}^2]/[2g\sigma_{b1}]\}}$	(First Stage Wheel Hub Thickness)
25.	σ_{b2}	$= 4.52\, A_2\, \rho_b (N/1000)^2$	(Last Stage Bucket Stress)
26.	F_2	$= f_5(T_{t2})$	(Last Stage Allowable Stress)
27.	t_{n2}	$= [\sigma_{b2} S]/[2 F_2]$	(Last Stage Wheel Neck Thickness)
28.	t_{h2}	$= t_{n2}\, e^{\{[\rho_b U_{h1}^2/[2 g \sigma_{b2}]\}}$	(Last Stage Wheel Hub Thickness)
29.	VV_2	$= [r_{t1} + r_{h1}]/[r_{h1}]$	(First Stage Pitch Line Velocity Ratio)
30.	CLR	$= [CL]/[r_{t1} - r_{h1}]$	(First Stage Clearance Ratio)
31.	η_{t1}	$= f_6 [V_1, M] - 3\,(CLR)_1$	(First Stage Efficiency)
32.	VV_2	$= [r_{t2} + r_{h2}]/[r_{h2} \operatorname{Sin} \alpha]$	(Last Stage Pitch Line Velocity Ratio)
33.	CLR_2	$= [CL]/[r_{t2} + r_{h2}]$	(Last Stage Clearance Ratio)
34.	η_{t2}	$= f_6 [V_2, M] - 3\,(CLR)^2$	(Last Stage Efficiency)
35.	η_t	$= [\eta_1 U_{h1}^2 + \eta_2 U_{h2}^2]/[U_{h1}^2 + U_{h2}^2]$	(Over-all Turbine Efficiency)

TABLE 7 (Continued)

Preliminary Turbine Sub-System Model

B. **Auxiliary Equations** (Continued)

36.	P	=	$W \Delta H_t \left[(\eta_t / .85) - LL \right]$	(Turbine Power Output)
37.	W_{b1}	=	$.5\ \pi \rho_b S\ (r_{t1}^2 - r_{h1}^2)$	(First Stage Bucket Weight)
38.	W_{r1}	=	$2/3\ \pi \rho_b r_{h1} S^2$	(First Stage Rim Weight)
39.	W_{w1}	=	$\pi/3\ \rho_b r_{h1}^2 \left[t_{h1} + 2\ t_{n1} \right] 1.2$	(First Stage Wheel Weight)
40.	W_{n1}	=	$\left[2\ \rho_s W_{b1} \right] / \rho_b$	(First Stage Nozzle Weight)
41.	W_{s1}	=	$\left[4 \pi \rho_s t_s \sqrt{\pi A_1} \left[r_{h1} + \sqrt{A_1 / \pi} \right] \right.$	(Inlet Scroll Weight)
42.	W_1	=	$W_{b1} + W_{r1} + W_{w1} + W_{n1}$	(First Stage Weight)
43.	W_{b2}	=	$.5\ \pi \rho_b S\ (r_{t2}^2 - r_{h2}^2)$	(Last Stage Bucket Weight)
44.	W_{r2}	=	$2/3\ \pi \rho_b r_{h2} S^2$	(Last Stage Rim Weight)
45.	W_{w2}	=	$\pi/3\ \rho_b r_{h2}^2 \left[t_{h2} + 2\ t_{n2} \right] 1.2$	(Last Stage Wheel Weight)
46.	W_{s2}	=	$4\ \pi \rho_s t_s \sqrt{\pi A_2} \left[r_{h2} + \sqrt{A_2 / \pi} \right]$	(Exit Scroll Weight)
47.	W_{n2}	=	$2\ \rho_s W_{b2} / \rho_b$	(Last Stage Nozzle Weight)
48.	W_2	=	$W_{b2} + W_{r2} + W_{w2} + W_{n2}$	(Last Stage Weight)
49.	W_c	=	$3/4\ \pi \rho_s t_c (r_{t1} + r_{t2} + t_c)\ L$	(Casing Weight)

C. **Constraints**

1. $\sigma_{b1} < F_1$

TABLE 7 (Continued)

Preliminary Turbine Sub-System Model

C. Constraints (Continued)

2. $\sigma_{b2} < F_2$

D. Objective

1. $W = 2W_{s1} + n/2\,(W_1 + W_2) + 2W_{s2} + W_c$

E. Parameters

Gas Constant - R

Density of Rotor Material - ρ_b (Molybdenum)

Density of Stator Material - ρ_s (Columbium)

Bucket Chord - S

Scroll Thickness - t_s

Casing Thickness - t_c

Specific Heat Ratio - γ, 1.22 for Potassium

F. Emperical Relations

1. $FF = f_1\,(V)$
2. $F_m = f_2$ (Working Fluid, T_{t1}, T_{t2})
3. $H_v = f_3$ (Working Fluid, T_{t1})
4. $P_{t2} = f_4\,(T_{t2})$
5. $F = f_5\,(T_t)$
6. $\eta_t = f_6\,(V_1, M)$

G. Auxiliary Outputs

Rotational Speed - N, RPM

Turbine Power Output - P, BTU/sec

TABLE 8

Preliminary Generator Sub-System Model
(Radial Gap Machine)

A. Input

1. Rotational Speed - N, RPM
2. Frequency - f, cps
3. Voltage - E
4. Gap Diameter - D_g, in
5. Gap Length - L_g, in
6. Flux Density Across Gap - B_g
7. Depth of Core Behind Slots - d, in
8. Slot Depth - S, in
9. Slot Pitch - p, in
10. Current Density - ID
11. Load Power Factor - PF
12. Coolant Temperature - T_c, $^{\circ}R$
13. Deep Bar Factor - F_t
14. Turbine Power Output - P, watts

B. Auxiliary Equations

1. $B_{tf} = .4\pi D_g L_g B_g$ (Frame Flux)
2. $D_c = \sqrt{4 B_{tf} / [\pi (B_r)_m]}$ (Core Diameter)

TABLE 8 (Continued)

Preliminary Generator Sub-System Model
(Radial Gap Machine)

B. **Auxiliary Equations** (Continued)

3.	g	=	$[D_g - D_c]/30$	(Gap)
4.	T	=	$60\ f/N$	(No. of Rotor Teeth)
5.	S	=	$\pi D_g/[6\ T p]$	(Slots/Pole/Phase)
6.	I	=	$.9\ P/[E\,(PF)]$	(Current)
7.	n	=	$[4\,(10)^{-8} f S K_d K_p B_{tf}]/[1.2\ E]$	(No. of Parallel Paths thru Winding)
8.	IS	=	$2\,I/n$	(Current per Slot)
9.	w	=	$[IS]/[\,ID(SSF)\ S]$	(Slot Width)
10.	B_{st}	=	$[B_g p\ (FF)]/[p - w]$	(Flux Density in Stator Teeth)
11.	B_{sc}	=	$B_{tf}/[2.4\ T d]$	(Flux Density in Stator Core)
12.	AT	=	$.626\ F_i\, g\, B_g + 1.35\,(IS)$	(Ampere Turns)
13.	l_f	=	$[2\,(AT)\,(FSF)\ /\,(IDD)$	(Field Coil Length)
14.	${}_r$	=	$\pi/4\ \rho_s (N/1000)^2 [1.507\,(D_g^2 - D_c^2)\,D_g/D_c + 1.230\,\pi D_c^2]$	(Rotor Stress)
15.	ρ	=	$f_1\,[T_c]$	(Resistivity)
16.	R_s	=	$[\,\rho F_t (2\ L_g + l_f + .55\,\pi\ D_g/T]/\,w_s (SSF)$	(Resistance per Slot)

TABLE 8 (Continued)

Preliminary Generator Sub-System Model
(Radial Gap Machine)

17. $P_c = 6TSR_s(IS)^2$ (Conductor I^2R loss)

18. $P_s = 2\pi\rho_s L_g K_m(FLT)^2\left\{sD_gB_{st}^2\left[\frac{p}{p-w}\right] + .25B_{se}^2\right.$

$\left.\left[(D_g + 2_s + 2d)^2 - (D_g + 2_s)^2\right]\right\}$ (Stator/Ion Loss)

19. If $P_c > P_s$, go to line 21

20. $P_{pf} = P_s$, go to line 22

21. $P_{pf} = P_c$ (Rotor Pole Face & Stray Load Loss)

22. $R_f = 2\pi\rho_c[D_g + 2_s + 2_d + .5l_f]/[(FSF)l_f^2]$ (Field Resistance)

23. $P_f = (AT)^2 R_f$ (Field Excitation Loss)

24. $PG = P - [P_c + P_s + P_{pf} + P_f]$ (Generator Power Output)

25. $EFF = PG / P$ (Generator Efficiency)

26. $W_{fr} = (2l_f + L_g)\rho_s \; B_{tf} / B_f$ (Frame Weight)

27. $W_{fc} = \pi / 2 \rho_c l_f^2 [D_g + 2_s + 2d + .5l_f]$

(Field Coil Weight)

28. $W_s = .55\pi\rho_s L_g[(D_g + 2_s + 2d)^2 - D_g^2]$

(Stack Weight)

29. $W_r = \pi / 4 \rho_s [D_c^2 (2l_g + l_f) + 2/3 L_g(D_g^2 - D_c^2)]$

(Rotor Weight)

TABLE 8 Continued

Preliminary Generator Sub-System Model

(Radial Gap Machine)

C. Constraints

1. $PG > 350{,}000$

2. $2\pi D_g > 90gT$

3. $B_{st} < (B_s)_m$

4. $B_{sc} \leq (B_s)_m$

5. $\dfrac{600\ K_c\ L_g\ D_g\ B_f}{d\ B_{tf}\ (D_g + 2d + 2s)^2} > P_e$

D. Objective

1. $W = W_{fr} + W_{fc} + W_s + W_r$

E. Parameters

Density of Copper - ρ_c

Density of Steel - ρ_s

Maximum Flux Density across Rotor - $(B_r)_m$

Maximum Flux Density across Stator - $(B_s)_m$

D. C. Current Density across Field Coil - IDD

(Frequency) (Lamination Thickness) - FLT

Flux Density in Frame - B_f

Fringing Factor - FF

Slot Space Factor - SSF

Field Space Factor - FSF

TABLE 8 (Continued)

Preliminary Generator Sub-System Model

(Radial Gap Machine)

Winding Constant - $K_d K_p$

Core Loss Proportion Constant - K_m

Ampere Turn Factor - F_i

F. Empirical Relations

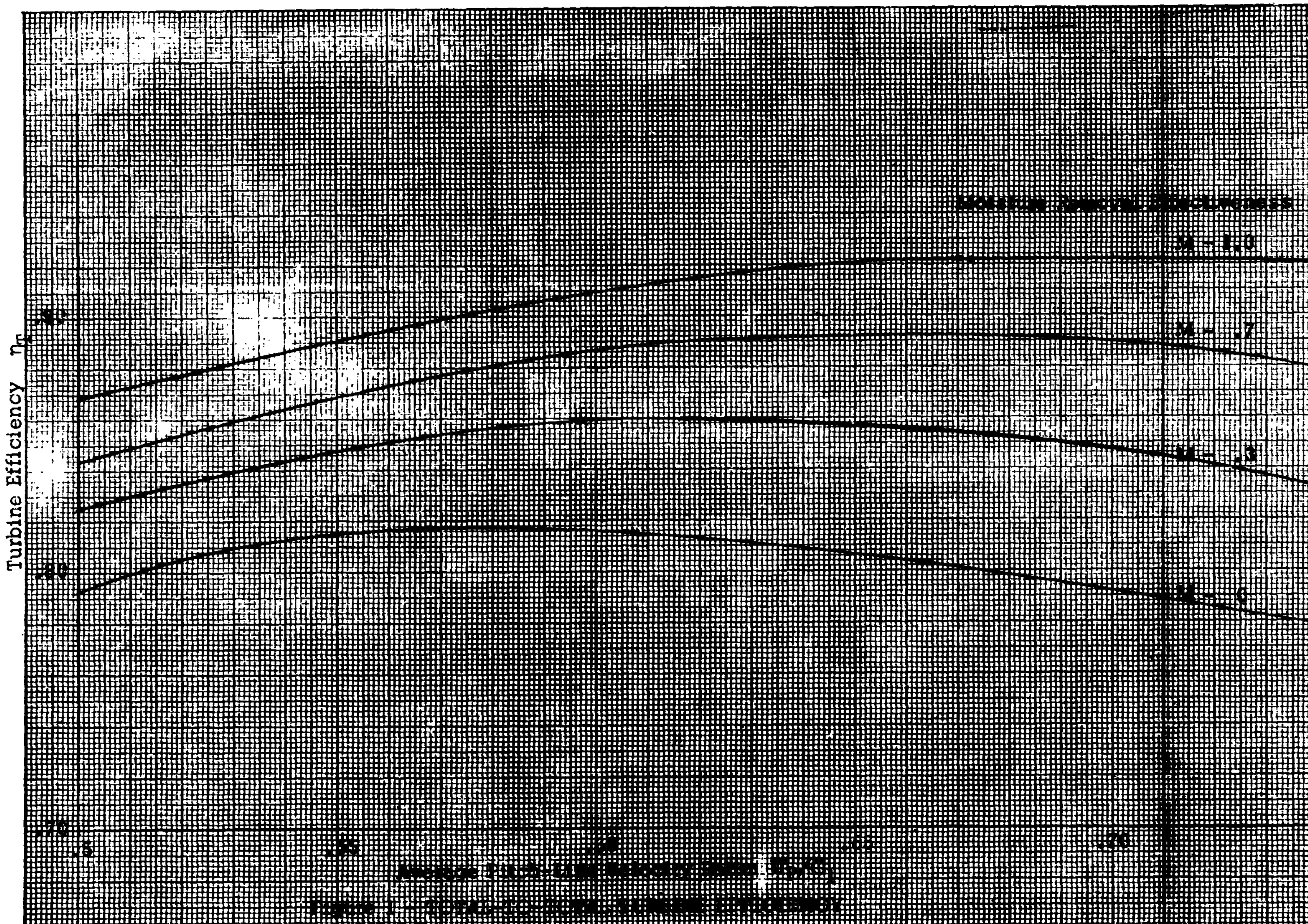
1. Resistivity - $\rho = f_1 \quad T_c$

G. Auxiliary Outputs

Current - I, amp.

Generator Power Output - PG, watts

Generator Efficiency - EFF


Turbine Efficiency η_T
M - 1.0
M - .7
M - .3
M - 0
.80
.70

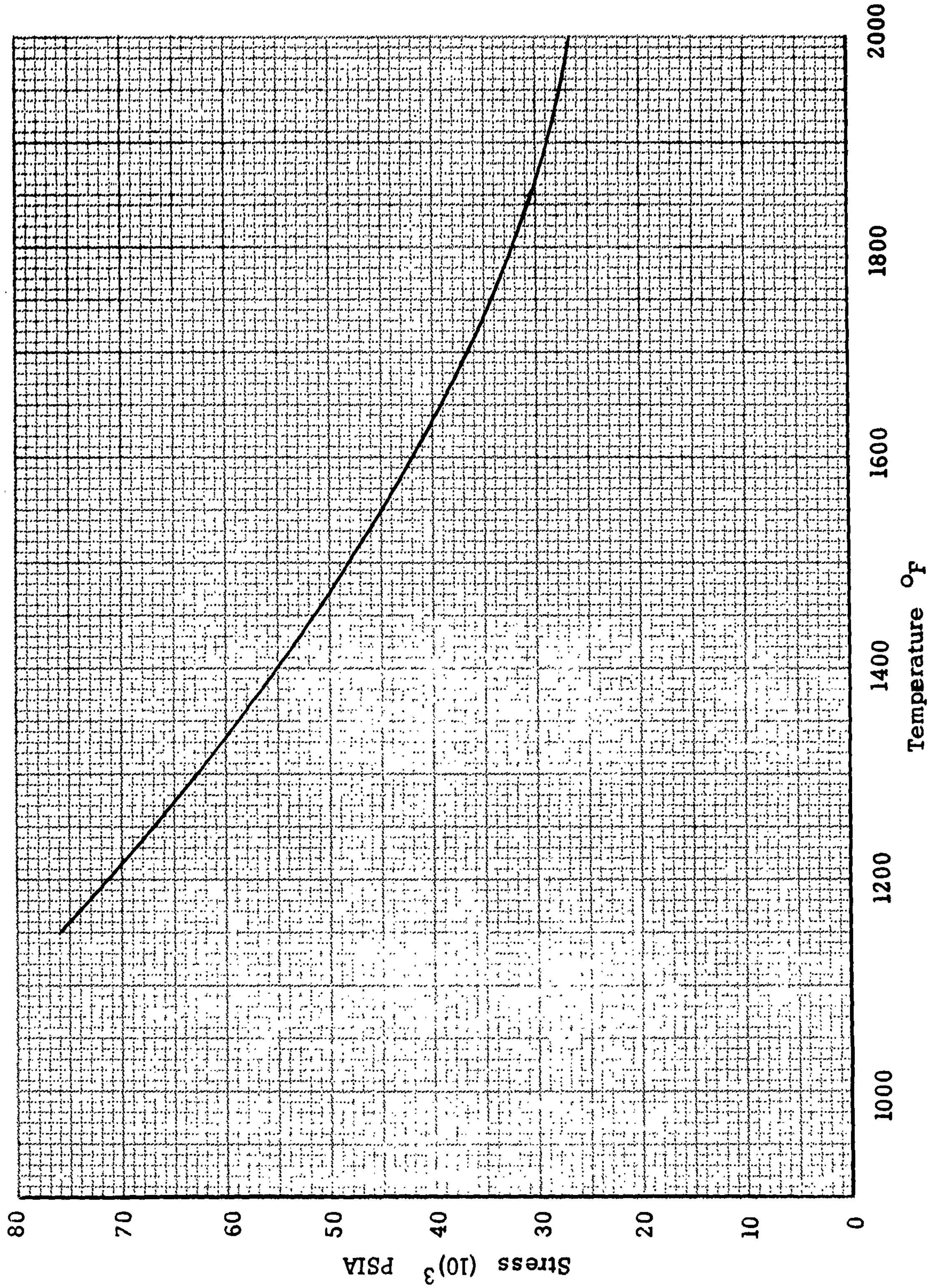
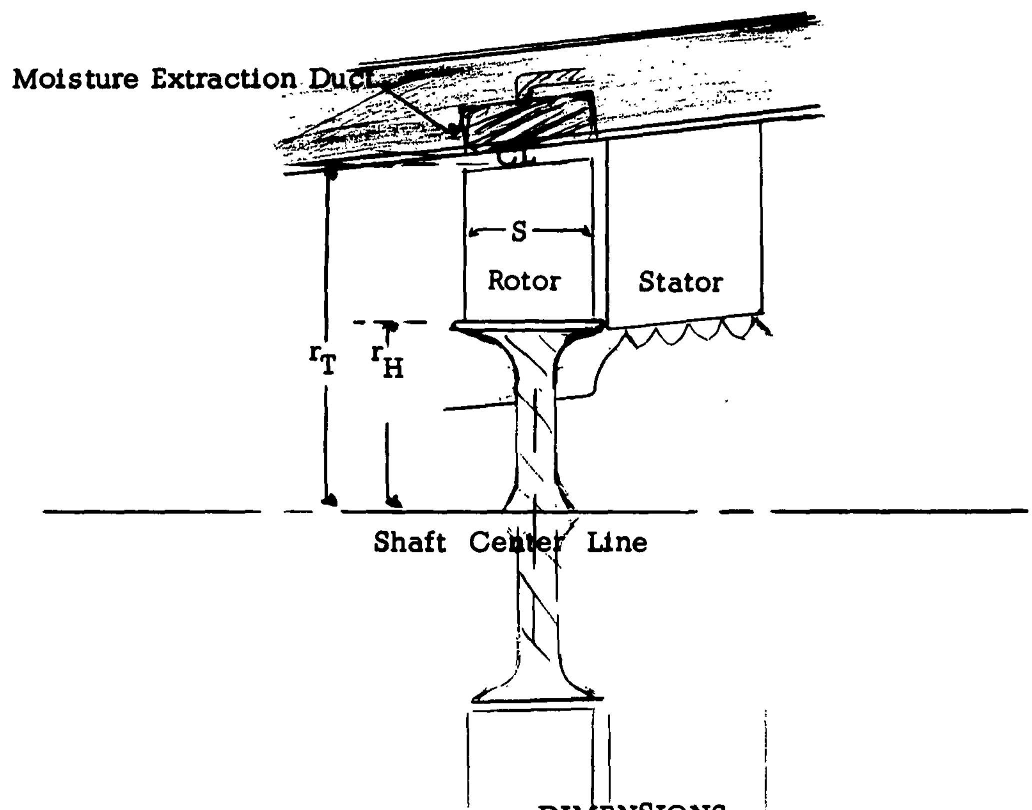


Figure 2 TURBINE ROTOR DESIGN STRESS LIMITS

PRELIMINARY TURBINE SUB-SYSTEM MODEL
(Multi-Stage Machine)



DIMENSIONS

CL - Clearance, in
r_T - Tip Radius, in
r_H - Hub Radius, in
S - Chord, in

Figure 3 TURBINE SCHEMA

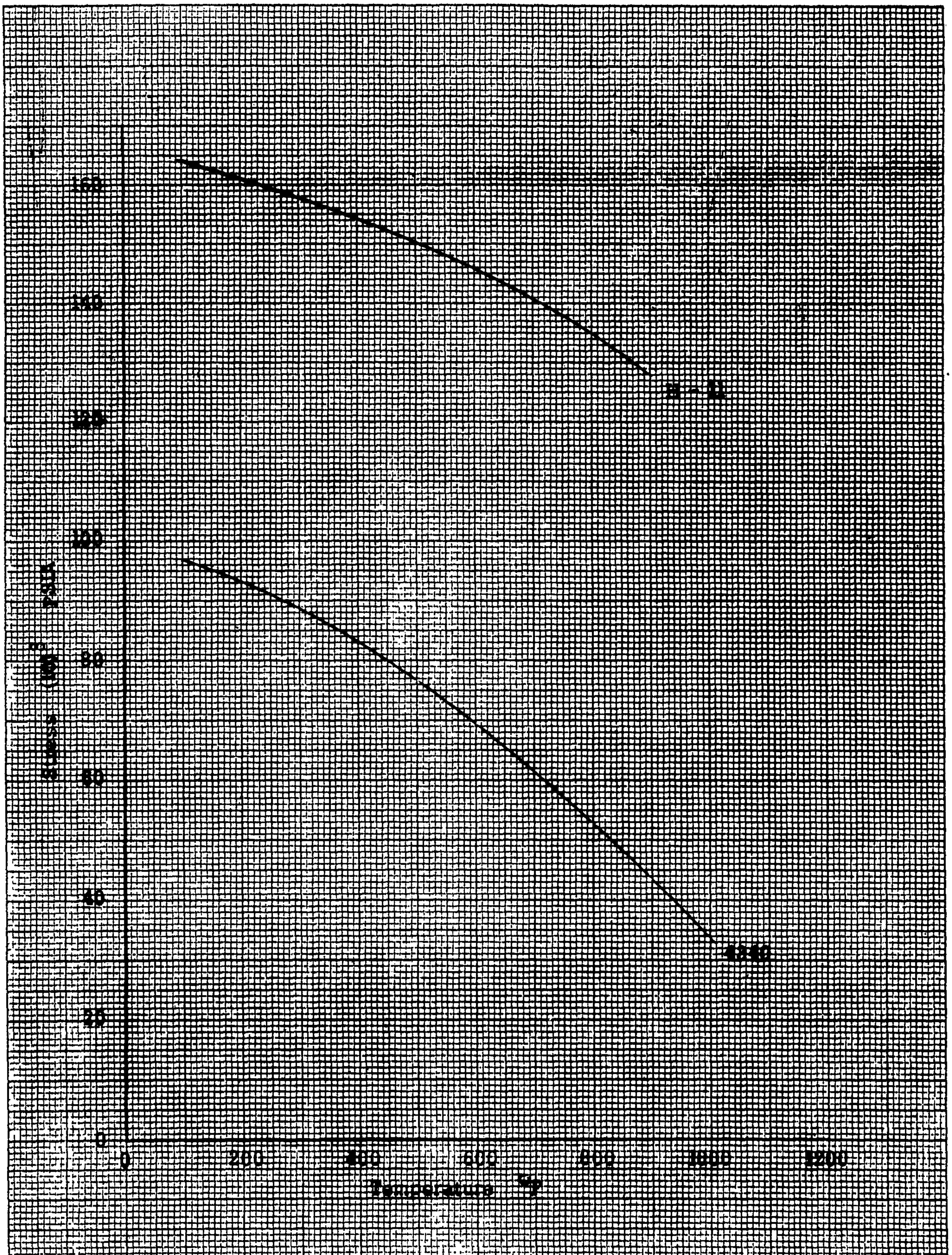


Figure 4 - GENERATOR ROTOR DESIGN STRESS LIMITS

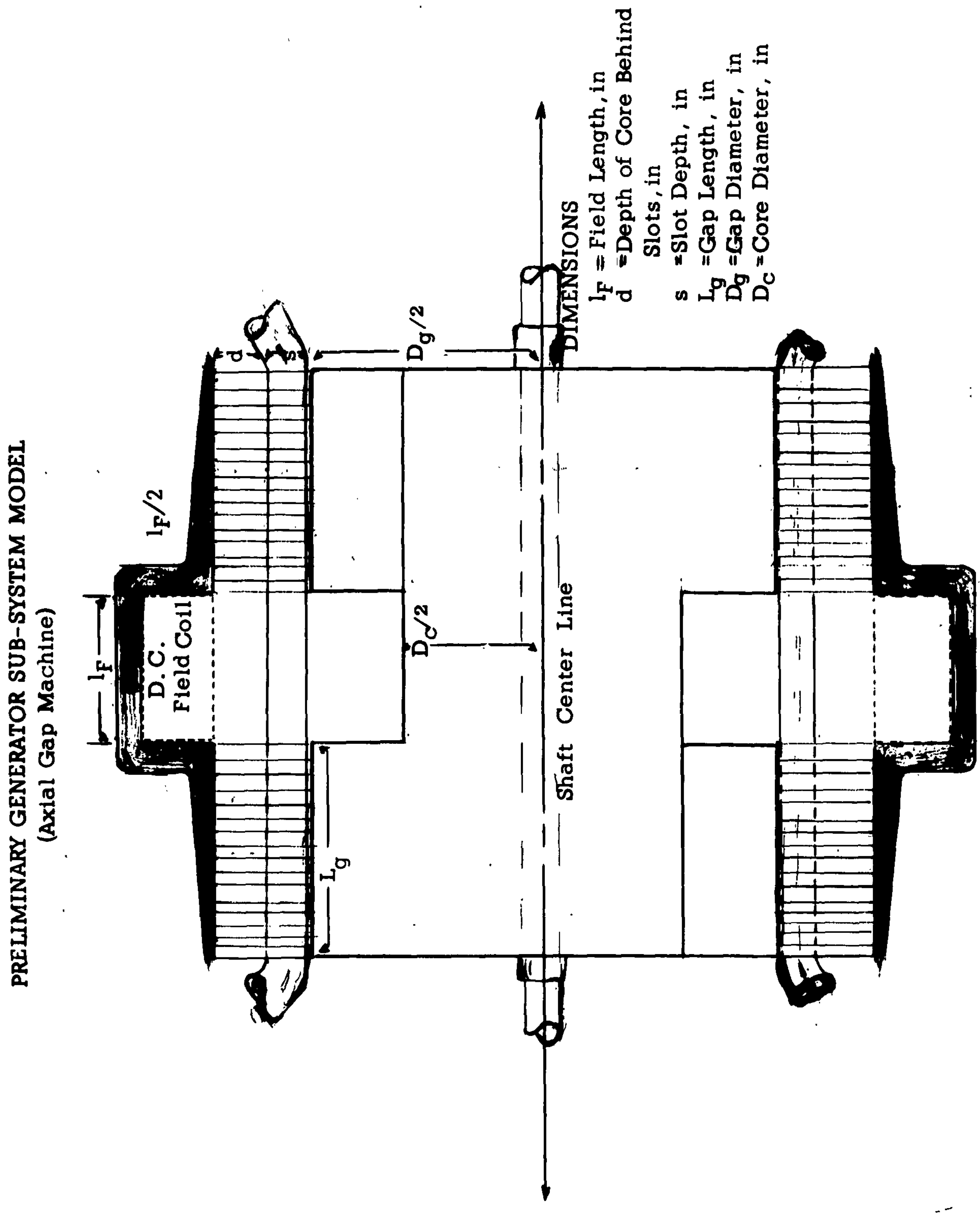


Figure 5 GENERATOR SCHEMA

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S U P P L E M E N T

<u>LEADER USER'S MANUAL</u>
(Model Development)

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LEADER USERS MANUAL

I. INTRODUCTION

The LEADER technique is the name applied by the General Electric Company to a procedure for determining the optimum values of the design variables of highly complex engineering designs. The procedure involves the development of an analytical model of the complete system which defines the interactions between each of the design variables and sub-systems and contains all desired design constraints and physical system limitations. The resulting model is then explored numerically, using the method of steepest descent, and an optimum design identified which is compatible with all of the imposed constraints.

The overall technique consists of two digital computer procedures - for model development and model optimization - and supplementary manual operations for the preparation of the input data required by the computer programs and for the evaluation of the results obtained. This manual contains a description of these manual operations as applied to a typical problem in order to facilitate the use of the LEADER technique. This manual is intended as a supplement to References (1) and (2) which describe the development of each of the computer programs, their format, and their listings. Such detail has, accordingly, been omitted from this manual.

A glossary of terms, nomenclature, and examples are included in the Appendices to this manual.

II. RESULTS ACHIEVABLE

The results of the model development phase of the LEADER technique are the reduction of the design characteristics of the system under investigation into a completely analytic form through the process of empirical approximation. The validity of the approximation is indicated through several "goodness of fit" criteria which can be used to substantiate the final results of the overall analysis. In addition, the relative importance of each of the variables and groupings of variables is indicated in order to add some additional insight into the characteristics of the overall system.

Those elements of the design which can be described analytically, need not be represented unless their analytical expressions are too complex to be handled straightforwardly. Thus, this phase would be concerned primarily with experimental data, graphical data, or statistical data.

The results of the system optimization phase are the values of each of the independent and dependent variables at the point which optimizes the design subject to the constraints specified. If the resulting design is constrained by any boundaries imposed upon the system, they will be identified. The sensitivity of the resulting design to small changes in each of the independent design variables will be indicated. Additional sensitivity information can sometimes be obtained from the path followed by the design in the iterative optimization procedure.

III. MODEL DEVELOPMENT PROCEDURES

The initial step of the model development process is the collection of all design procedures, experimental data, graphical data, etc. which define the design problem, the interactions between all of the significant parameters, and the physical constraints or boundaries imposed upon the system. In order to facilitate the subsequent analysis, the system should be broken down into sub-systems or smaller component parts. All elements which can be described in analytical form should be put aside and reserved for the optimization process. All remaining non-analytical elements must be reduced to some type of analytical form.

The procedure to be used in developing the analytical forms for the remaining elements will be dependent upon whether 1, 2, or more independent variables are involved. It is desirable, therefore, to subdivide the elements into the smallest elements which will describe each physical effect so that the task of empirical approximation will be minimized. Each of these procedures is described in the following sections.

A. One Independent Variable

Those elements which can be described in terms of a single-line graph or a single-entry table can be empirically represented by the procedure contained in this section. The result will be an empirical equation of the form:

$$y = B_0 + B_1 x + B_2 x^2 + B_3 x^3 + B_4 x^4 \tag{1}$$

where the polynomial form is utilized in order to achieve results which are most easily compatible with the optimization procedures of the following section.

The initial step is to select the range of variation in x over which the dependent variable - y - is to be fitted and to determine the degree of approximation which will be required in order to achieve a satisfactory representation. The independent variable is normalized over the range of 0 to 1 by the transformation equations:

$$a = (x)_{min}$$

$$b = (x)_{max} \tag{2}$$

$$c = \frac{1}{b-a}$$

$$X = c(x-a)$$

If the data is in tabular form, it will be necessary to plot it before proceeding. A degree of polynomial representation is then selected according to the shape of the data. If the data is non-linear, for example, at least the second degree should be selected. From the data of Table 1a, specific values of X are noted at which corresponding values of y must be obtained from its curve. Table 1b is then utilized to calculate the probable fitting error associated with the degree of polynomial representation selected.

$$\epsilon = C_o y_o + C_1 y_1 + C_2 y_2 + C_3 y_3 + \ldots \tag{3}$$

Errors of the order of 1 to 2 % are normally acceptable. If the calculated error is considered unacceptable, the process should be repeated with the next higher degree of representation. If the use of the fourth degree is still unable to produce acceptable representation it will be necessary to either transform the variables or to reduce the fitting region.

Transformations such as $\ln x$, $\ln y$, $1/x$, $1/y$, $\sin x$, $\sin y$, etc. may be utilized in place of the original x,y values in equations (1), (2), and (3). Unfortunately, no specific guidelines can be offered as to the transformations that will work for any one problem. The user must apply his own judgment in this area. If the transformation approach does not produce acceptable results, however, the fitting accuracy may be improved by reducing the range - $1/c$ - over which the fit is to be obtained.

After an acceptable degree of fit has been obtained, the data of Table 2 is used to obtain the actual coefficients of the fit. For a second degree fit, for example, the first coefficient A_0 is obtained from the equation:

$$A_0 = .8333\ldots y_0 + .333\ldots y_1 - .333\ldots y_2 + .166\ldots y_3 \qquad (4)$$

Similarly, the values of A_1 and A_2 are obtained from the other lines of the second degree table. The resulting equation will express the dependent variable in terms of the normalized independent variable:

$$y = A_0 + A_1 X + A_2 X^2 + \ldots A_n X^n \qquad (5)$$

Equation (5) must then be unnormalized to reduce it to the form of equation (1). This can be accomplished by calculating columns 2 through 9 of Table 3. The first unnormalized coefficient is then obtained from a summation of the products of column 4 with column 5:

$$B_o = A_o - acA_1 + a^2c^2A_2 - a^3c^3a_3 + a^4c^4A_4 \tag{6}$$

Similarly, the other coefficients are obtained from the product of column 4 with each of the remaining columns 6 through 9.

The resulting equation can then be used directly in the optimization process. The curve fitting technique employed minimizes the amount of manual effort required for obtaining the required coefficients. It is described in more detail in Reference (3). If, however, computer curve fitting programs are available, they may be used in place of the above procedure.

B. Two Independent Variables

Those elements of the system which can be described in terms of a family of curves or a double-entry table can be represented by a procedure which is an extension of the one described in Section A. The result will be an empirical equation of the form:

$$\begin{aligned} y = {} & B_{00} + B_{01}\, x + B_{02} + \dots B_{0n}\, x^n \\ & + B_{10}z + B_{11}\, xz + B_{12}x^2z + \dots B_{1n}x^nz \\ & + B_{20}z^2 + B_{21}xz^2 + B_{22}x^2z + \dots B_{2n}x^2z^2 \\ & + \dots \end{aligned} \tag{7}$$

The initial step is to determine the range of variation required in x and z and to determine the degree of approximation required for each variable. Note that the degree in x need not be the same as the degree in z. Both variables must be normalized over the 0 to 1 range by equation (2).

Table 1 is then used to determine fitting errors in both the x and z directions. Transformation techniques must be considered at this point if desirable fitting accuracies are not obtained.

After the required degree of fit in each direction has been determined, the coefficients are determined in a manner quite similar to that used in the uni-variate fitting process described in the previous section. The bi-variate fitting is, in fact, accomplished by performing a series of uni-variate fittings. The initial data table will consist of the following information:

	X_o	X_1	X_2	X_3	X_4	X_5
Z_o	y_{00}	y_{01}	y_{02}	y_{03}	y_{04}	y_{05}
Z_1	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}
Z_2	y_{20}	y_{21}	y_{22}	y_{23}	y_{24}	y_{25}
Z_3	y_{30}	y_{31}	y_{32}	y_{33}	y_{34}	y_{35}
Z_4	y_{40}	y_{41}	y_{42}	y_{43}	y_{44}	y_{45}
Z_5	y_{50}	y_{51}	y_{52}	y_{53}	y_{54}	y_{55}

(8)

The first fitting consists of the representation of y_o as a function of X at $Z = Z_o$. The result is an equation of the form:

$$y_o = A_{00} + A_{01} X + A_{02} X^2 + A_{03} X^3 + A_{04} X^4 \text{ at } Z = Z_o \tag{9}$$

This process is repeated at Z_1, Z_2, Z_3, Z_4, and Z_5. The resulting data can then be written as:

$$\begin{array}{c|ccccc} & X^0 & X^1 & X^2 & X^3 & X^4 \\ \hline Z_0 & A_{00} & A_{01} & A_{02} & A_{03} & A_{04} \\ Z_1 & A_{10} & A_{11} & A_{12} & A_{13} & A_{14} \\ Z_2 & A_{20} & A_{21} & A_{22} & A_{23} & A_{24} \\ Z_3 & A_{30} & A_{31} & A_{32} & A_{33} & A_{34} \\ Z_4 & A_{40} & A_{41} & A_{42} & A_{43} & A_{44} \\ Z_5 & A_{50} & A_{51} & A_{52} & A_{53} & A_{54} \end{array} \tag{10}$$

The resulting coefficients are then cross-fit in terms of powers of Z in a similar fashion. Thus, the first column of line (1) results in an equation of the form:

$$A_o = \left[D_{00} + D_{10} Z + D_{20} Z^2 + D_{30} Z^3 + D_{40} Z^4\right] X^0 \tag{11}$$

This process is repeated for each of the other columns. The result is an equation of the form:

$$\begin{aligned} y = {} & D_{00} + D_{01} X + D_{02} X^2 + D_{03} X^3 + D_{04} X^4 \\ & + D_{10} Z + D_{11} XZ + D_{12} X^2 Z + D_{13} X^3 Z + D_{14} X^4 Z \\ & + D_{20} Z^2 + D_{21} XZ^2 + D_{22} X^2 Z^2 + D_{23} X^3 Z^2 + D_{24} X^4 Z^2 \\ & + D_{30} Z^3 + D_{31} XZ^3 + D_{32} X^2 Z^3 + D_{33} X^3 Z^3 + D_{34} X^4 Z^3 \\ & + D_{40} Z^4 + D_{41} XZ^4 + D_{42} X^2 Z^4 + D_{43} X^3 Z^4 + D_{44} X^4 Z^4 \end{aligned} \tag{12}$$

The resulting equation (12) must then be unnormalized to the form of equation (7). The process involves a series of linear unnormalizing steps, each of which are in accordance with the procedure of Table 3.

C. Three or More Independent Variables

Those elements of the system which are functions of three or more variables can be represented by the procedure contained in this section. The result will be an empirical equation of the form:

$$\begin{aligned} y = B_{00} &+ B_{01} x_1 + B_{02} x_2 + B_{03} x_3 + \ldots + B_{0n} x_n \\ &+ B_{12} x_1 x_2 + B_{13} x_1 x_3 + \ldots + B_{1n} x_1 x_n \\ &\qquad + B_{23} x_2 x_3 + \ldots + B_{2n} x_2 x_n \\ &\qquad\qquad + \ldots \\ &+ B_1 x_1^2 + B_2 x_2^2 + B_3 x_3^2 + \ldots + B_n x_n^2 \end{aligned} \tag{13}$$

where x_1 through x_n are the independent variables involved. Note that no terms of order three or greater are included. The absence of these higher order terms will make the fitting procedure more dependent upon the user's ability to identify the proper format or transformation to be used for each variable.

Although no specific guidelines can be offered to aid in the identification of the proper transformations, the use of approximate analyses may be extremely helpful. If ineffective transformations are used for any of the variables, however, this factor will be indicated by the results of the fitting operation. The offender could then be replaced by a more suitable variable transformation and a new fitting obtained. The techniques of section A may be helpful in determining which transforms can be represented with sufficient accuracy with second-order fittings in each variable.

After the variables and their transforms have been determined, an experiment plan must be selected to match the number of variables involved. The number of coefficients to be determined in developing equation (13) is equal to:

$$\text{Number of Coefficients} = 1 + 2n + \frac{n!}{2(n-2)!} \tag{14}$$

The number of data points to be used in determining the coefficients can be found from the equation:

$$\text{Number of Data Points: } N = 2^{n-p} + 2n + k \tag{15}$$

where n is the number of variables, p is the order of replication (0 for full replicate, 1 for half, 2 for quarter, etc.), and k is the number of center-points. The resulting degrees of freedom can be obtained by subtracting equation (14) from equation (15). Table 4 contains a summary of these data for plans involving a single center-point.

All plans must have at least one degree of freedom and should, in general, have 5 to 10 degrees of freedom in order to maintain a balance between the number of points required and the number of terms confounded by the partial replication process. Consequently, the following plans are recommended:

Full Replicate for n up to 4

Half Replicate for n of 5 and 6

Quarter Replicate for n of 7

Eighth Replicate for n of 8

The orthogonal factor - α - must then be calculated from the equation:

$$\alpha = \left[\frac{1}{2} \left\{ \sqrt{N(2)^{n-p}} - (2)^{n-p} \right\} \right]^{\frac{1}{2}} \tag{16}$$

Each of the independent variables must then be normalized over the range of -1 to + 1. This is accomplished by the following transform equations:

$$\begin{aligned} a &= (x)_{min.} \\ b &= (x)_{max.} \\ x &= \left(\frac{b-a}{2}\right) X + \left(\frac{b+a}{2}\right) \end{aligned} \tag{17}$$

The unnormalized independent variables must then be calculated, using the last of equations (17), at:

$$\begin{aligned} X &= -\alpha \\ &= -1 \\ &= 0 \\ &= 1 \\ &= \alpha \end{aligned} \tag{18}$$

The specific experiment plan to be used must then be obtained from Table 5 which lists the specific combinations of independent variables at which the dependent variable is to be evaluated. Table 5 also lists the first order interaction terms which are confounded by the partial replication process. It is desirable to assign independent variable numbers so that only marginally significant effects are confounded. Additional experiment plans are available from Reference (4).

The dependent variable must then be evaluated at all levels indicated by the experiment plan. The results of line (18) will assist in converting the coded values of the independent variables in the experiment plan to actual values.

The next step in the procedure will be to transfer this information to the computer input sheet which will contain the necessary instructions for the key punch operator. Some additional information will be required and this will be indicated where necessary. A sample input sheet is illustrated in Table 6.

Section A contains information which will serve to identify the specific problem, the investigator, and the date of calculation.

Section B contains the type of fractional replication selected from Table 4.

Section C contains the factorial part of the experiment plan selected. It may, however, be omitted when a full replicate is used. Line

3X1 contains the coded values of the first independent variable - the + 1 or - 1 values. If more than 22 values are required, the remainder must be continued on a second card as illustrated. Similarly, 3X2 describes the second independent variable, 3X3 the third, etc. Note that it is not necessary to indicate the cross values ($\pm \alpha$) or center-points.

Section D contains the values of the dependent variable corresponding to the experiment plan selected and to the sequence used in Section C. All factorial, cross, and center-point values must be included. The first card starts with the identification 3Y1 and contains the first 9 values. Each succeeding card is started in the second column and contains the next 10 values until all data have been included.

Section E contains the F-ratio selected from Table 7 corresponding to the number of degrees of freedom associated with the experiment plan and the probability that the deviations observed could occur by chance alone. It is recommended that probabilities of 1 to 5 % should normally be selected.

Section F contains the boundaries on each of the independent variables. Line 3Z1 contains the uncoded minimum value of the first variable and the maximum value. Similarly, line 3Z2 is for the second, 3Z3 for the third, etc. Note that these values must correspond to the - 1 and + 1 coded values.

Section G contains control information as follows:

M is the number of independent variables (10 maximum)

N is the number of data points

NC is the number of center-points (10 maximum)

NR is the number of multiple replications (normally 1)

NP is the order of fractional replication (0 for full, 1 for half, etc.)

AN is zero if analysis of variance is desired, otherwise one.

RSOP is zero if response surface optimization is not desired, otherwise one.

HIGH is zero if higher order interactions are not desired, otherwise one.

CONF is zero if confidence intervals are not desired, otherwise one.

TPTS is zero if the unnormalized model equation is not desired, otherwise one.

LOGE is zero if a model based upon log y is not desired, otherwise one.

ACT should be identical with TPTS

It is recommended that the following values be used normally,

NC = 1	CONF = 1
NR = 1	TPTS = 1

AN = 0 LOG E = 0

RSOP = 0 ACT = 1

HIGH = 0

The input sheet can then be processed and the data run on the computer. Results should be checked for key punch errors by comparing the print-out against the input sheet. The results should then be evaluated to determine if the fit is of sufficient accuracy for use in the subsequent system optimization. The following data will be printed:

1. The uncoded values of each of the independent variables at the $-\alpha$, -1, 0, 1, and α levels. These data should correspond with manual calculations using equation (18).

2. The experiment plan which shows the coded values of all independent variables at each of the data points. These data should correspond with the plan selected and with the data of section C of Table 6.

3. The dependent variable table which should correspond to section D of Table 6.

4. The comparison of observed and calculated values of the dependent variable at each of the input data points. In addition to a direct comparison, several measures of error are available. The most significant of these is the term labeled PCT/RNGE which is determined from the formula:

$$\text{PCT/RNGE} = \frac{\text{Calculated y} - \text{Observed y}}{(\text{Observed y})_{\text{max.}} - (\text{Observed y})_{\text{min.}}} \times 100 \qquad (19)$$

This measure indicates the efficiency of the fitting operation since a maximum of 50 % could have been obtained by representing the model as a constant. Although some judgment is needed in determining an acceptable error criteria the following guidlines may be used:

0 to 2 % is an excellent fit

2 % to 10 % may be an acceptable fit

Above 10 % is generally a poor fit

If a poor fit is obtained with correct input data, this may be remedied by using an alternate transformation for one or more of the independent variables. The variables which should be transformed are those which are at their $\pm\alpha$ values at the points where the largest errors are obtained. No specific guidelines can be offered, however, to assist in determining the proper transformation to be used.

5. The variance of the estimate and coefficients. The term labeled standard error of the estimate is, in actuality, the variance of the estimate and is a measure of the dispersion of the individual observations about their mean. It represents another measure of the goodness of fit of the derived analytical model.

6. The analysis of variance. This tabulation contains the F-ratio associated with each of the derived coefficients of the model. The highest F-ratio corresponds to the most significant term in the model and provides a means for ranking the individual terms in order of their significance.

All terms which have an F-ratio less than the value in section E of Table 6 can normally be ignored.

7. The coefficients of the model in terms of the coded values of the independent variables. The number in parenthesis indicates whether or not the coefficient is a significant one within the prescribed value of F-ratio - 1 indicates the term is significant and 0 that it is not.

8. A repeat of line 4 using only the significant terms. If the measures of error are significantly higher than those obtained in line 4, the selected F-ratio is probably too high and some other value should be used.

9. The coefficients of the model in terms of the uncoded values of the independent variables including both significant and insignificant terms. Insignificant terms can be eliminated by referring to the results of line 7. This should be done with caution since the unnormalizing process will result in contributions to the linear terms from the quadratic terms. The results of this last step may be utilized directly in the subsequent system optimization process.

IV. REFERENCES

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3. "Scientific Computation Forum - Proceedings of 1948", by H.R.J. Grosch: Editor IBMC, New York 22, New York

4. "Experimental Designs", 2nd Edition - 1957, by W.G. Cochran and C.M. Cox John Wiley and Sons, New York

V. GLOSSARY

Full Factorial Design - An array of n independent variables (factors) at 2, 3, or more coded levels of each variable in which all combinations of coded levels and factors are used simultaneously to establish the characteristics of a dependent variable. The number of treatments or combinations required is 2^n, 3^n, etc. All feasible linear effects or combinations of variables can be determined independently.

Fractional Factorial Design - An array in which only 1/2, 1/4, etc. of the full factorial design is used to establish the characteristics of the dependent variable. The number of treatments required is equal to 2^{n-p}, 3^{n-p}, etc. where p is the order of replication and is equal to 0 for a full replicate, 1 for a half replicate, 2 for a quarter replicate, etc. It is used to reduce the number of treatments required.

Box-Wilson Factorial Design - A modified fractional factorial design in which the 2^{n-p} treatments are increased by k center points at the center of the region and 2 n cross points at $\pm \alpha$ values of each factor successively. The additional treatments permit the estimation of quadratic terms for each variable.

Confounding - The process by which the linear effects are grouped in order to permit estimation with a fractional factorial design.

Alias Pairs - Specific effects which are confounded by the partial replication or fractional factorial design. Each alias pair will consist of two effects with a half replicate, three with a quarter replicate, etc.

Defining Contrast - The pattern which identifies the specific treatments retained by a fractional replicate and the effects which are confounded.

Degrees of Freedom - The difference between the total number of data points or treatments used and the number of coefficients which are calculated.

Variance - The mean value of the sum of squares of the deviations of a number of observations about their mean.

Standard Deviation - The square root of the variance.

Objective Function - The variable that is to be optimized subject to the constraints provided.

Constraints - Linear or non-linear boundaries placed upon the independent variables in the optimization process.

Table 1

Selection of the Degree of Polynomial Representation

a. **Independent Variable Values**

Degree	0	1	2	3	4
X_0	0	0	0	0	0
X_1	1.000	.500	.250	.150	.095
X_2		1.000	.750	.500	.345
X_3			1.000	.850	.654
X_4				1.000	.904
X_5					1.000

b. **Error Coefficients**

Degree	0	1	2	3	4
C_0	+.5	+.25	+.166...	+.125	+.10
C_1	-.5	-.50	-.333...	-.250	-.20
C_2		+.25	+.333...	+.250	+.20
C_3			-.166...	-.250	-.20
C_4				+.125	+.20
C_5					-.10

Table 2

Curve Fitting with Chebyshev Polynomials

Zero Degree Polynomial

a. Equation

$$y = A_o X^o \qquad \& \ X = \frac{x - x_{min.}}{x_{max} - x_{min.}}$$

b. Intervals

x_o	x_1
0	1

c. Coefficients

	y_o	y_1
A_o	+.5	+.5
ϵ =	+ 1/2	- 1/2

Table 2 (Continued)

First Degree Polynomial

a. Equation

$$y = A_o X^o + A_1 X^1 \quad \& \; X = \frac{x - x_{min.}}{x_{max.} - x_{min.}}$$

b. Intervals

x_o	x_1	x_2
0	.5	1

c. Coefficients

	y_o	y_1	y_2
A_o =	+.75	+.5	-.25
A_1 =	- 1	0	+ 1
ϵ =	+ 1/4	- 1/2	+ 1/4

Table 2 (Continued)

Second Degree Polynomial

a. Equation

$$y = A_o X^o + A_1 X^1 + A_2 X^2 \quad \& \quad X = \frac{x - x_{min.}}{x_{max.} - x_{min.}}$$

b. Intervals

x_o	x_1	x_2	x_3
0	.25	.75	1

c. Coefficients

	y_o	y_1	y_2	y_3
A_o =	+ .83333.....	+ .3333.....	- .3333.....	+ .16666.....
A_1 =	- 3.3333.....	+2	+ 3.3333.....	-2
A_2 =	+ 2.6666.....	- 2.6666.....	- 2.6666.....	+ 2.6666.....
ϵ =	+ 1/6	-1/3	+1/3	-1/6

Table 2 (Continued)

Third Degree Polynomial

a. Equation

$$y = A_o X^o + A_1 X^1 + A_2 X^2 + A_3 X^3 \quad \& \quad X = \frac{x - x_{min.}}{x_{max.} - x_{min.}}$$

b. Intervals

X_o	X_1	X_2	X_3	X_4
0	.14644661	.5	.85355339	1

c. Coefficients

	y_o	y_1	y_2	y_3	y_4
A_o =	+ .875	+ .25	- .25	+ .25	- .125
A_1 =	- 7	+ 5.6568542	+ 4	- 5.6568542	+ 3
A_2 =	+ 14	- 16.970563	- 4	+ 16.970563	- 10
A_3 =	- 8	+ 11.313708	0	- 11.313708	+ 8
ϵ =	+ 1/8	- 1/4	+ 1/4	- 1/4	+ 1/8

Table 2 (Continued)

Fourth Degree Polynomial

a. Equation

$$y = A_o X^o + A_1 X^1 + A_2 X^2 + A_3 X^3 + A_4 X^4 \quad \& \quad X = \frac{x - x_{min.}}{x_{max.} - x_{min.}}$$

b. Intervals

x_o	x_1	x_2	x_3	x_4	x_5
0	.09549	.65451	.65451	.90451	1

c. Coefficients

	y_o	y_1	y_2	y_3	y_4	y_5
A_o =	+ .9	+ .2	- .2	+ .2	- .2	+ .1
A_1 =	- 12	+ 10.9442719	+ 4.21114562	- 6.9442719	+ 7.7885438	- 4
A_2 =	+ 43.2	- 56.7213596	+ 1.65510699	+ 32.7213596	- 44.85510699	+ 24
A_3 =	- 57.6	+ 86.7987578	- 21.2879228	- 41.9987578	+ 78.8879228	- 44.8
A_4 =	+ 25.6	- 41.42167014	+ 15.82167014	+ 15.82167014	- 41.42167014	+ 25.6
ϵ =	+ 1/10	- 1/5	+ 1/5	- 1/5	+ 1/5	- 1/10

Table 3

Unnormalizing Procedure

1	2	3	4					
i	c^i	A_i	$c^i A_i$	5				
0	1	A_0	A_0	1	6			
1	c	A_1	cA_1	$-a$	1	7		
2	c^2	A_2	$c^2 A_2$	$+a^2$	$-2a$	1	8	
3	c^3	A_3	$c^3 A_3$	$-a^3$	$+3a^2$	$-3a$	1	9
4	c^4	A_4	$c^4 A_4$	$+a^4$	$-4a^3$	$+6a^2$	$-4a$	1

Table 4

Solution of Experiment Plan

		Full		Half		Quarter		Eighth	
Variables	Coefficients	Points	d.f.	Points	d.f.	Points	d.f.	Points	d.f.
1	3	5	2						
2	6	9	3						
3	10	15	5						
4	15	25	10						
5	21	43	22	27	6				
6	28	77	49	45	17	29	1		
7	36			79	43	47	11		
8	45					81	36	49	4
9	55					147	92	83	28
10	66							149	83

Table 5

<u>DESIGN MATRIX PLAN</u>

1. Number of independent variables - $n = 3$
2. Full Replication ($p = 0$)
3. Number of data points - $N = (2^{n-p} + 2\,n + K)$ where: K = number of center points
4. Alpha value - α is obtained from:

$$\alpha = 1/2\ (\ 2^{n-p}\ (2^{n-p} + 2n + K)\ -\ 2^{n-p}p\ \ ^{1/2}$$

	N Number of Observations	Factor Level X_1	X_2	X_3
Full Factorial Portion	1	1	1	1
	2	1	1	-1
	3	1	-1	1
	4	1	-1	-1
	5	-1	1	1
	6	-1	1	-1
	7	-1	-1	1
	8	-1	-1	-1
6 Alpha Values	9	α	0	0
	10	$-\alpha$	0	0
	11	0	α	0
	12	0	$-\alpha$	0
	13	0	0	α
	14	0	0	$-\alpha$
Center Point	15	0	0	0

Table 5 - (Continued)

DESIGN MATRIX PLAN

1. Number of independent variables - $n = 4$
2. Full Replication $(p = 0)$
3. Number of data points - $N = (2^{n-p} + 2n + K)$ where: K = number of center points
4. Alpha value - α is obtained from:

$$\alpha = 1/2 \left(2^{n-p} (2^{n-p} + 2n + K) - 2^{n-p} \right)^{1/2}$$

	N Number of Observations	Factor Level X_1	X_2	X_3	X_4
Full Factorial Portion	1	1	1	1	1
	2	1	1	1	-1
	3	1	1	-1	1
	4	1	1	-1	-1
	5	1	-1	1	1
	6	1	-1	1	-1
	7	1	-1	-1	1
	8	1	-1	-1	-1
	9	-1	1	1	1
	10	-1	1	1	-1
	11	-1	1	-1	1
	12	-1	1	-1	-1
	13	-1	-1	1	1
	14	-1	-1	1	-1
	15	-1	-1	-1	1
	16	-1	-1	-1	-1
8 Alpha Points	17	α	0	0	0
	18	$-\alpha$	0	0	0
	19	0	α	0	0
	20	0	$-\alpha$	0	0
	21	0	0	α	0
	22	0	0	$-\alpha$	0
	23	0	0	0	α
	24	0	0	0	$-\alpha$
Center Point	25	0	0	0	0

Table 5 - (Continued)

DESIGN MATRIX PLAN

1. Number of independent variables - $n = 5$
2. Half Replication ($p = 1$)
3. Number of data points - $N = (2^{n-p} + 2n + K)$ where: K = number of center points
4. Alpha value - α is obtained from:

$$\alpha = 1/2\ (\ 2^{n-p}\ (2^{n-p} + 2\,n + K - 2^{n-p})^{1/2}$$

	N Number of Observations	Factor Level X_1	X_2	X_3	X_4	X_5
Half Factorial Portion	1	1	1	1	1	-1
	2	1	1	1	-1	1
	3	1	1	-1	1	1
	4	1	1	-1	-1	-1
	5	1	-1	1	1	1
	6	1	-1	1	-1	-1
	7	1	-1	-1	1	-1
	8	1	-1	-1	-1	1
	9	-1	1	1	1	1
	10	-1	1	1	-1	-1
	11	-1	1	-1	1	-1
	12	-1	1	-1	-1	1
	13	-1	-1	1	1	-1
	14	-1	-1	1	-1	1
	15	-1	-1	-1	1	1
	16	-1	-1	-1	-1	-1
10 Alpha Values	17	α	0	0	0	0
	18	$-\alpha$	0	0	0	0
	19	0	α	0	0	0
	20	0	$-\alpha$	0	0	0
	21	0	0	α	0	0
	22	0	0	$-\alpha$	0	0
	23	0	0	0	α	0
	24	0	0	0	$-\alpha$	0
	25	0	0	0	0	α
	26	0	0	0	0	$-\alpha$
Center Point	27	0	0	0	0	0

Defining Contrast: $X_1 X_2 X_3 X_4 X_5$

Table 5 - (Continued)

DESIGN MATRIX PLAN

1. Number of variables - $n = 6$
2. Half Replication ($p = 1$)
3. Number of data points - $N = (2^{n-p} + 2n + K)$ where: K = number of center points
4. Alpha value - α is obtained from:

$$\alpha = 1/2\,(\;2^{n-p}\,(2^{n-p} + 2\,n + K) - 2^{n-p})^{\;1/2}$$

	N Number of Observations	Factor Level X_1	X_2	X_3	X_4	X_5	X_6
	1	-1	-1	-1	-1	-1	-1
	2	1	1	-1	-1	1	1
	3	1	-1	1	1	1	-1
	4	-1	1	1	1	-1	1
	5	1	1	-1	-1	-1	-1
	6	-1	-1	-1	-1	1	1
	7	1	-1	1	1	-1	1
	8	-1	1	1	1	1	-1
	9	1	-1	1	-1	-1	-1
	10	-1	-1	-1	1	1	-1
	11	1	1	-1	1	-1	1
	12	-1	1	1	-1	1	1
	13	-1	1	1	-1	-1	-1
	14	-1	-1	-1	1	-1	1
	15	1	-1	1	-1	1	1
Half Factorial Portion	16	1	1	-1	1	1	-1
	17	1	-1	-1	-1	1	-1
	18	-1	1	-1	-1	-1	1
	19	-1	-1	1	1	-1	-1
	20	1	1	1	1	1	1
	21	1	-1	-1	-1	-1	1
	22	-1	1	-1	-1	1	-1
	23	1	1	1	1	-1	-1
	24	-1	-1	1	1	1	1
	25	1	-1	-1	1	-1	-1
	26	-1	-1	1	-1	1	-1
	27	1	1	1	-1	-1	1
	28	-1	1	-1	1	1	1
	29	-1	1	-1	1	-1	-1
	30	-1	-1	1	-1	-1	1
	31	1	1	1	-1	1	-1
	32	1	-1	-1	1	1	1

Continued on next page

Table 5 - (Continued)

DESIGN MATRIX PLAN - Continued

1. Number of variables - n = 6
2. Half Replication (p = 1)

	N Number of Observations	X_1	X_2	X_3	X_4	X_5	X_6
		Factor Level					
12 Alpha Values	33	α	0	0	0	0	0
	34	$-\alpha$	0	0	0	0	0
	35	0	α	0	0	0	0
	36	0	$-\alpha$	0	0	0	0
	37	0	0	α	0	0	0
	38	0	0	$-\alpha$	0	0	0
	39	0	0	0	α	0	0
	40	0	0	0	$-\alpha$	0	0
	41	0	0	0	0	α	0
	42	0	0	0	0	$-\alpha$	0
	43	0	0	0	0	0	α
	44	0	0	0	0	0	$-\alpha$
Center Point	45	0	0	0	0	0	0

Defining Contrast: $X_1 X_2 X_3 X_4 X_5 X_6$

Table 5 - (Continued)

DESIGN MATRIX PLAN

1. Number of variables - $n = 7$
2. Quarter Replication ($p = 2$)
3. Number of data points - $N = (2^{n-p} + 2n + K)$ where: K = number of center points
4. Alpha value - α is obtained from:

$$\alpha = 1/2 \left(2^{n-p} (2^{n-p} + 2n + K) - 2^{n-p} \right)^{1/2}$$

	N Number of Observations	Factor Level X_1	X_2	X_3	X_4	X_5	X_6	X_7
	1	-1	-1	-1	-1	-1	-1	-1
	2	-1	1	1	-1	-1	-1	-1
	3	1	-1	-1	1	-1	1	-1
	4	1	-1	-1	-1	1	-1	1
	5	-1	-1	-1	1	1	1	1
	6	1	1	1	1	-1	1	-1
	7	1	1	1	-1	1	-1	1
	8	-1	1	1	1	1	1	1
	9	-1	1	-1	1	-1	-1	1
	10	-1	1	-1	-1	1	1	-1
	11	-1	-1	1	-1	1	1	-1
	12	1	1	-1	-1	-1	1	1
	13	1	-1	1	-1	-1	1	1
	14	1	1	-1	1	1	-1	-1
	15	1	-1	1	1	1	-1	-1
Quarter Factorial Portion	16	-1	-1	1	1	-1	-1	1
	17	1	1	-1	-1	-1	-1	-1
	18	1	-1	1	-1	-1	-1	-1
	19	-1	1	-1	1	-1	1	-1
	20	-1	1	-1	-1	1	-1	1
	21	-1	-1	1	1	-1	1	-1
	22	-1	-1	1	-1	1	-1	1
	23	1	-1	1	1	1	1	1
	24	1	1	-1	1	1	1	1
	25	-1	-1	-1	1	1	-1	-1
	26	-1	-1	-1	-1	-1	1	1
	27	1	-1	-1	1	-1	-1	1
	28	1	-1	-1	-1	1	1	-1
	29	-1	1	1	1	1	-1	-1
	30	-1	1	1	-1	-1	1	1
	31	1	1	1	1	-1	-1	1
	32	1	1	1	-1	1	1	-1

Continued on next page

Table 5 - (Continued)

DESIGN MATRIX PLAN - Continued

1. Number of variables - n = 7
2. Quarter Replication (p = 2)

	N Number of Observations	X_1	X_2	X_3	X_4	X_5	X_6	X_7
14 Alpha Values	33	α	0	0	0	0	0	0
	34	$-\alpha$	0	0	0	0	0	0
	35	0	α	0	0	0	0	0
	36	0	$-\alpha$	0	0	0	0	0
	37	0	0	α	0	0	0	0
	38	0	0	$-\alpha$	0	0	0	0
	39	0	0	0	α	0	0	0
	40	0	0	0	$-\alpha$	0	0	0
	41	0	0	0	0	α	0	0
	42	0	0	0	0	$-\alpha$	0	0
	43	0	0	0	0	0	α	0
	44	0	0	0	0	0	$-\alpha$	0
	45	0	0	0	0	0	0	α
	46	0	0	0	0	0	0	$-\alpha$
Center Point	47	0	0	0	0	0	0	0

Factor Level: X_1 through X_7

Defining Contrasts: $X_1X_2X_3X_4X_5$

$X_1X_2X_3X_6X_7$

$X_4X_5X_6X_7$

$X_4X_6 = X_5X_7$, $X_4X_5 = X_5X_7$, $X_4X_7 = X_5X_6$ - Alias Pairs

Table 6

Box Design Data Sheet (L0749B)

Section A

(Title Card, begin all punching in Column 1)

5C9D, 10, 1 ____________________

Section B

(Data, Replication ________, Begin punching in Column 2)

Section C

(Values of Y, FACTORIAL or FRACTIONAL FACTORIAL Part - Continue on second card, Column 2, if more than 72 columns are required for each independent variable)

3X1, ____________________

____

3X2, ____________________

____

3X3, ____________________

____

3X4, ____________________

____

3X5, ____________________

____

3X6, ____________________

____

3X7, ____________________

____

Table 6. (Continued)

Box Design Data Sheet (LO749B)

Section D

(Values of Y, Cross Polytopes, $+\alpha_1$, $-\alpha_1$, $+\alpha_2$, $-\alpha_2$, etc. - Continue on second card, Column 2, if more than 72 columns are required for the N_Y Observed Values)

3Y1, ____________________

Section E

(Value of F RATIO TEST Level)

3W1, ________

Section F

3Z1, ________, ________,
3Z2, ________, ________,
3Z3, ________, ________,
3Z4, ________, ________,
3Z5, ________, ________,
3Z6, ________, ________,
3Z7, ________, ________,

Section G

(Control Card)

→ M, ___ M, ___ N, ___ NC, ___ NR, ___ NP, ___ AN, ___ RSOP, ___ HIGH, ___ CONF, ___ TPTS, ___ LOGC, ___ ACT,

TABLE 7

F-Ratio

Degrees of Freedom	Probability .05	.01	.001
1	161.4	4052	405,284
2	18.51	98.50	998.5
3	10.13	34.12	167.5
4	7.709	21.20	74.14
5	6.608	16.26	47.04
6	5.987	13.74	35.51
7	5.591	12.25	29.22
8	5.318	11.26	25.42
9	5.117	10.56	22.86
10	4.965	10.04	21.04
11	4.844	9.646	19.69
12	4.747	9.330	18.64
13	4.667	9.074	17.81
14	4.600	8.862	17.14
15	4.543	8.683	16.59
16	4.494	8.531	16.12
17	4.451	8.400	15.72
18	4.414	8.285	15.38
19	4.381	8.185	15.08
20	4.351	8.096	14.82
21	4.325	8.017	14.59
22	4.301	7.945	14.38
23	4.279	7.881	14.19
24	4.260	7.823	14.03
25	4.242	7.770	13.88